TRINITY COLLEGE

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

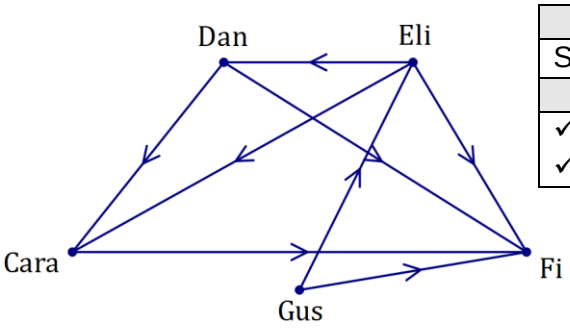
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) In a group of five people it was known that Eli was older than Cara, Dan and Fi; Dan was older than Cara and Fi; Gus was older than Eli and Fi; and Cara was older than Fi.

(i) Represent this set of age relationships as a digraph. (2 marks)



Solution
See graph
Specific behaviours
✓ all arcs
✓ all directions correct

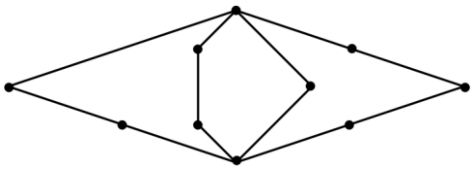
(ii) State the number of arcs in the digraph. (1 mark)

Solution
8 arcs
Specific behaviours
✓ correct number

(iii) List the five people in order of age, starting with the youngest. (1 mark)

Solution
Fi, Cara, Dan, Eli, Gus
Specific behaviours
✓ correct order

(b) Graph H is shown below.



Let t and p be the number of edges in the longest open trail and shortest closed path contained in H respectively. State the values of t and p, given that $t > 0$ and $p > 0$. (2 marks)

Solution
$t = 11, \quad p = 5$
Specific behaviours
✓ correct value of t
✓ correct value of p

See next page

Question 10 **(8 marks)**

The following data shows the blood haemoglobin (H) levels and packed cell volumes (V) of 10 blood bank donors.

H	13.5	16.3	14.4	17.7	18.1	14.9	15.2	13.8	16.9	14.2
V	0.36	0.39	0.37	0.41	0.42	0.38	0.40	0.37	0.41	0.38

(a) Graph the data on your calculator and describe features of the graph that suggest the presence of a strong and positive linear association between H and V. (2 marks)

Solution
Points lie very close to a straight line (strong, linear) As one variable increases, the other variable also tends to increase (positive)
Specific behaviours
✓ describes one feature ✓ describes all three features (strong, linear, positive)

(b) Determine the equation of the least-squares line that models the relationship between H and V, where H is the explanatory variable. Use four decimal places. (2 marks)

Solution
$V = 0.0115H + 0.210$
Specific behaviours
✓ states correct equation ✓ coefficients to at least 4dp (gradient) and 2dp (intercept)

(c) Determine the correlation coefficient between H and V. (1 mark)

Solution
$r = 0.939$
Specific behaviours
✓ correct value, at least 2dp

(d) What percentage of the variation in V can be explained by the variation in H? (1 mark)

Solution
88%
Specific behaviours
✓ correct to nearest whole number

(e) Predict the packed cell volume of a donor with a blood haemoglobin level of 15.6. (1 mark)

Solution
$\hat{V}(15.6) = 0.39$
Specific behaviours
✓ correct prediction

(f) Describe a potential danger associated with using the least-squares line to predict a packed cell volume from a blood haemoglobin level. (1 mark)

Solution
A prediction that involves extrapolation is dangerous and likely to be unreliable.
Specific behaviours
✓ mentions extrapolation

See next page

Question 11 **(7 marks)**

A company bought and installed a new computer system with an initial value of $\$13\,200$. For accounting purposes, the value of the system decreases by $\$880$ each year.

(a) Calculate the value of the system after 2 years. (1 mark)

Solution
$13200 - 2 \times 880 = \$11\,440$
Specific behaviours
✓ correct value

(b) Determine a recurrence relation for V_n, the value of the system after n years. (2 marks)

Solution
$V_{n+1} = V_n - 880,\ V_0 = 13200$
Specific behaviours
✓ recurrence relation ✓ shows a term of sequence

(c) Determine

(i) the value of the system after 9 years. (1 mark)

Solution
$V_9 = \$5\,280$
Specific behaviours
✓ correct value

(ii) the number of years for the value of the system to become nothing. (1 mark)

Solution
$V_n = 0 \Rightarrow n = 15$
Specific behaviours
✓ correct time

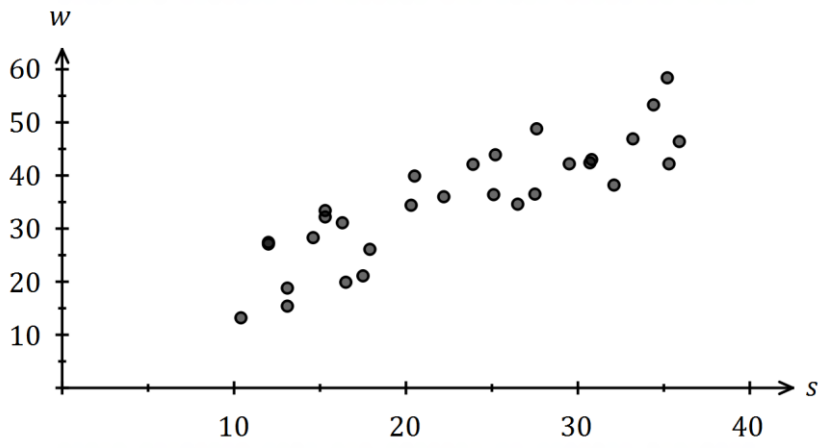
(d) Determine the decrease in time taken for the system to become worthless if its value decreases by $\$1\,100$ each year instead of $\$880$. (2 marks)

Solution
$V_{n+1} = V_n - 1100,\ V_0 = 13200$ $V_n = 0 \Rightarrow n = 12$ $15 - 12 = 3$ fewer years
Specific behaviours
✓ new total time ✓ states decrease in time

See next page

Question 12 **(7 marks)**

The scatterplot below shows the marks scored by 30 students in their spoken (s) and written (w) exams that were marked out of 50 and 70 marks respectively.



The equation of the least-squares line for the data is $w = 1.16s + 8.6$.

(a) It was found that 74% of the variation in w could be explained by the variation in s. Determine the correlation coefficient r_{sw}. (1 mark)

Solution
$r_{sw} = \sqrt{0.74} = 0.86$
Specific behaviours
✓ correct r to at least 2dp

(b) Interpret the slope of the least-squares line. (2 marks)

Solution
For every extra spoken mark, the corresponding written mark tends to increase by 1.16.
Specific behaviours
✓ identifies increase ✓ uses 1.16

(c) Lee and May were absent for the written exam, but it was known that their marks in the spoken exam were 41 and 22 respectively. Predict their written exam marks and explain how reliable each prediction is. (4 marks)

Solution
Lee: $w(41) = 56$. Lee's predicted score of 56 is unreliable as despite strong correlation it is extrapolated. May: $w(22) = 34$. May's predicted score of 34 is reliable as correlation is strong and it is interpolated.
Specific behaviours
✓ both predictions ✓ comments at least once on strong correlation ✓ uses extrapolation to justify Lee's is unreliable ✓ uses interpolation to justify May's is reliable

See next page

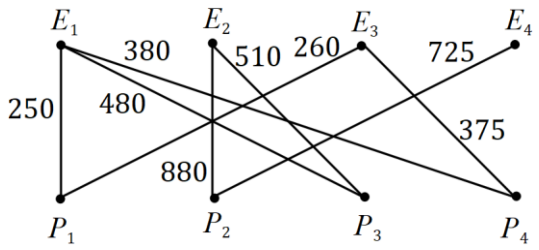
Question 13 **(8 marks)**

A builder has quotes from four electricians (E) to carry out repairs at four properties (P). The quotes are in dollars and not all electricians quoted for all properties, as shown in the table below.

($)	P_1	P_2	P_3	P_4
E_1	250	–	480	380
E_2	–	880	510	–
E_3	260	–	–	375
E_4	–	725	–	–

(a) Draw a weighted bipartite graph to represent this information. (4 marks)

Solution



Specific behaviours

✓ row of electricians and row of properties, both labelled
✓ correct number of edges from each E vertex
✓ correct number of edges from each P vertex
✓ adds costs to all edges

The builder decides to give all the electricians one property each to repair.

(b) Calculate the total cost to repair all four properties if E_3 repaired P_1. (2 marks)

Solution

$$E_1P_4 + E_2P_3 + E_3P_1 + E_4P_2$$
$$= 380 + 510 + 260 + 725 = \$1\,875$$

Specific behaviours

✓ indicates correct pairings
✓ correct total

(c) Determine the minimum total repair cost and the allocation of electricians to achieve this minimum. (2 marks)

Solution

$$E_1P_1 + E_2P_3 + E_3P_4 + E_4P_2$$
$$= 250 + 510 + 375 + 725 = \$1\,860$$

Specific behaviours

✓ states allocation
✓ correct minimum cost

See next page

Question 14 **(11 marks)**

A researcher obtained the following data whilst investigating whether it is possible to reliably predict a child's reading ability (A, on a numerical scale of 1 to 20) from their hand span (H, cm).

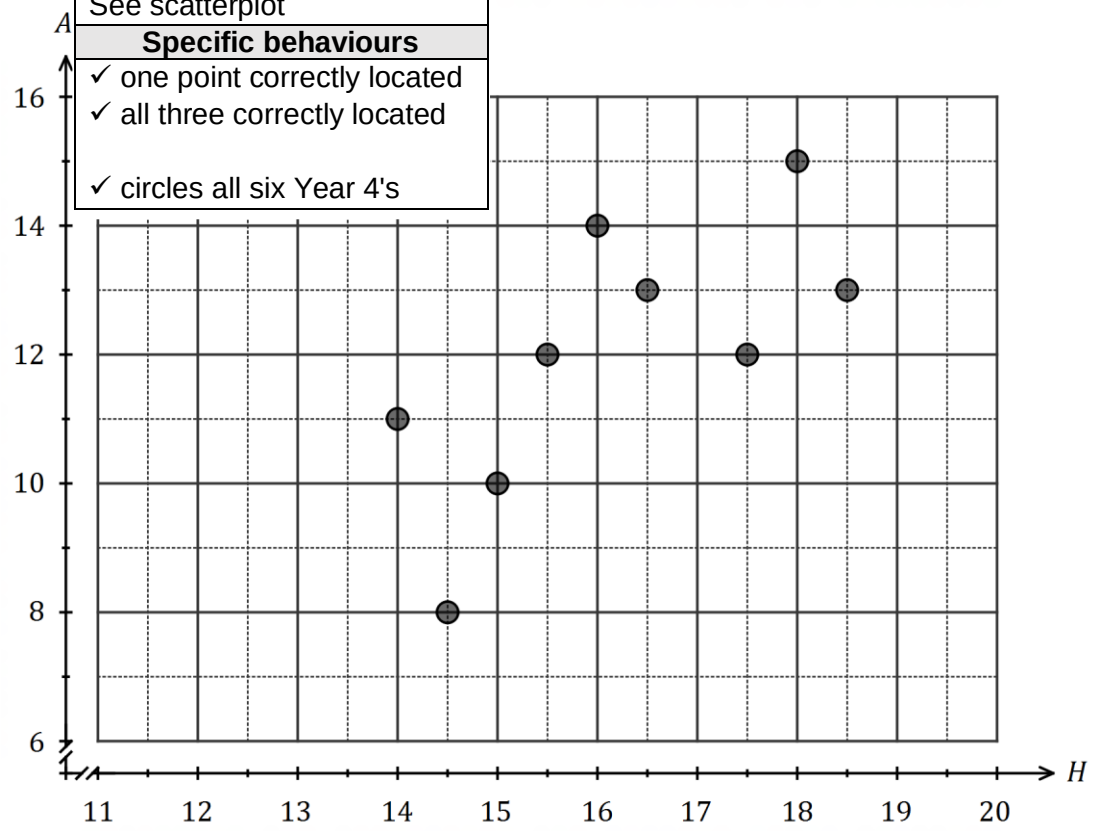
Child	B	C	D	E	F	G	J	K	L	M	N	P
H	16.5	15.5	15.0	14.0	13.0	18.5	17.5	13.5	12.5	14.5	16.0	18.0
A	13	12	10	11	8	13	12	7	9	8	14	15

(a) State the explanatory variable for this investigation. (1 mark)

Solution
Hand span
Specific behaviours
✓ correct variable

(b) Add the three missing data points to the scatterplot below. (2 marks)

Solutions (b) and (e)
See scatterplot
Specific behaviours
✓ one point correctly located
✓ all three correctly located
✓ circles all six Year 4's



(c) Determine the correlation coefficient between the two variables. (1 mark)

Solution
$r = 0.824$
Specific behaviours
✓ correct value (at least 2dp)

See next page

(d) Using the scatterplot from (b) and the correlation coefficient from (c), the researcher was satisfied that a linear associated existed between A and H. Explain why they reached this conclusion. (2 marks)

Solution
The points lie close to a straight line, indicating linear form. The correlation coefficient is close to 1, indicating a strong association.
Specific behaviours
✓ explains linear form ✓ refers to strong correlation

The researcher then discovered that the children labelled B, C, G, J, N and P were all in Year 8 and the remainder in Year 5.

(e) Circle the Year 8 children on the graph. (1 mark)

(f) Calculate the correlation coefficient between A and H for the Year 8 children only. (1 mark)

Solution
$r = 0.289$
Specific behaviours
✓ correct value (at least 2dp)

(g) Identify a non-causal explanation for the conclusion reached by the researcher in (d) and explain how this new information affects that conclusion. (3 marks)

Solution
A non-causal explanation is that the year group of a child is a confounding variable. The new information indicates there is no association as within a year group the correlation is very weak.
Specific behaviours
✓ identifies year group as a confounding variable ✓ states there is no association ✓ uses weak correlation to justify no association

See next page

Question 15 **(7 marks)**

A water tank is initially empty. At the start of each hour, 150 L of water is quickly poured into the tank but during the following hour, 20% of all the water in the tank leaks out.

This situation can be modelled by the recurrence relation $V_{n+1} = 0.8V_n + 150,\ V_0 = 150$, where V_n is the volume of water in the tank, in litres, at the start of the n^{th} hour.

(a) Complete the table below, giving volumes to the nearest litre. (2 marks)

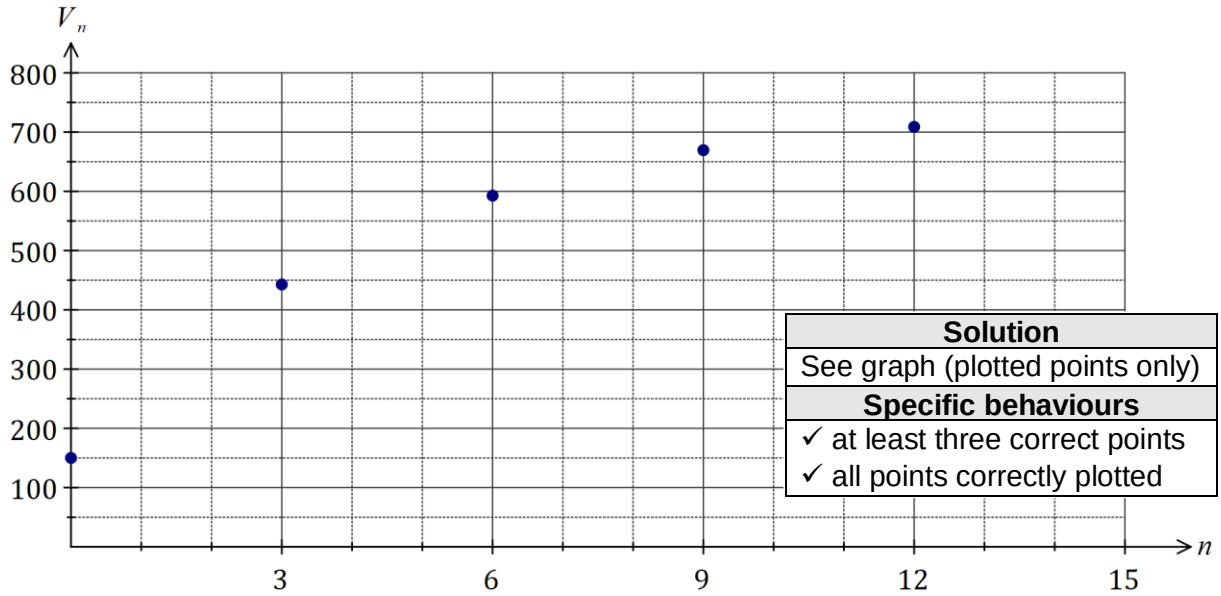
n	0	3	6	9	12
V_n	150	443	593	669	

Solution
See table
Specific behaviours
✓ at least two correct values
✓ all correct values

(b) At the start of which hour does the tank first hold at least 745 L? (1 mark)

Solution
$V_n \geq 745 \Rightarrow n = 22$
At the start of the 22nd hour.
Specific behaviours
✓ correct number

(c) Plot the points from the table on the axes below. (2 marks)



Solution
See graph (plotted points only)
Specific behaviours
✓ at least three correct points
✓ all points correctly plotted

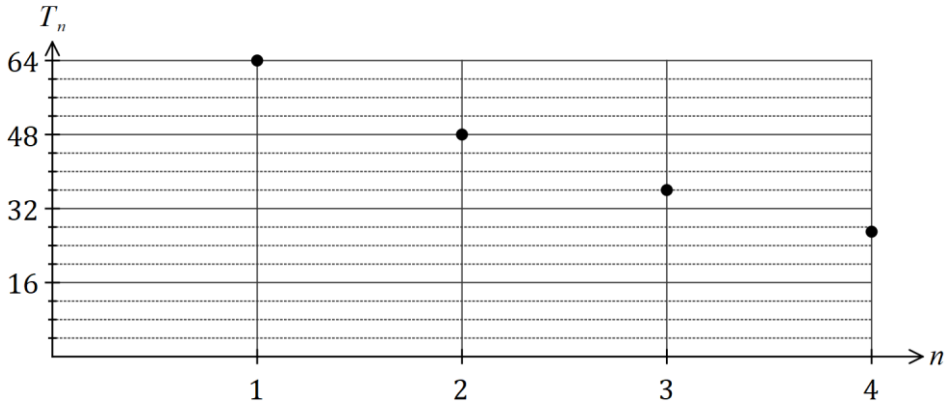
(d) The tank has a maximum capacity of 800 L. If possible, determine the least number of hours since filling commenced that the tank will start to overflow. If not possible, explain why not. (2 marks)

Solution
Not possible as the amount of water in the tank will never exceed 750 L.
Specific behaviours
✓ states not possible
✓ uses value of steady-state maximum

See next page

Question 16 **(6 marks)**

A piledriver is hammering a pile into the ground. The graph below shows the distance T_n (in cm) the pile moves into the ground on the n^{th} hit of the piledriver.



The values of T_n form a geometric sequence.

(a) Use information from the graph to determine the common ratio for the sequence. (1 mark)

Solution
$r = 48 \div 64 = 0.75$
Specific behaviours
✓ correct ratio

(b) Write a recurrence relation to generate the values of T_n. (2 marks)

Solution
$T_{n+1} = T_n \times 0.75, \quad T_1 = 64$
Specific behaviours
✓ recurrence relation ✓ states value of T_1

(c) Write the n^{th} term rule for the values of T_n. (1 mark)

Solution
$T_n = 64(0.75)^{n-1}$
Specific behaviours
✓ correct rule

(d) Determine

(i) the distance the pile moves into the ground on the tenth hit of the piledriver. (1 mark)

Solution
$T_{10} = 4.8$ cm
Specific behaviours
✓ correct distance

(ii) on which hit the pile first moves less than one mm into the ground. (1 mark)

Solution
$T_n < 0.1 \Rightarrow n = 24$
Specific behaviours
✓ correct value

Question 17 **(9 marks)**

A study categorised the weight of hospitalised children as underweight, normal, overweight or obese. The numbers of children in each category are shown by gender in the table below.

	Underweight	Normal	Overweight	Obese
Female	30	195	52	29
Male	12	174	81	17

(a) An obese child is randomly chosen from the study. If possible, explain whether they are more likely to be male or female. If not possible, explain your reasoning. (2 marks)

Solution
More likely to be a girl, as the number of obese girls is almost twice the number of obese boys.
Specific behaviours
✓ chooses girls ✓ explanation that compares numbers

(b) What percentage of the males in the study were classified as underweight? (2 marks)

Solution (b)
$\frac{12}{284} = 4.2\%$
Specific behaviours
✓ correct fraction ✓ correct percentage

Solution (c)
See table
Specific behaviours
✓ two entries correct ✓ one row correct ✓ all entries correct

(c) Complete the table of **row** percentages below to the nearest whole number. (3 marks)

(%)	Underweight	Normal	Overweight	Obese
Female	10	64	17	9
Male	4	61	29	6

(d) Does the table of row percentages suggest the presence of an association between the categorical variables? Justify your answer. (2 marks)

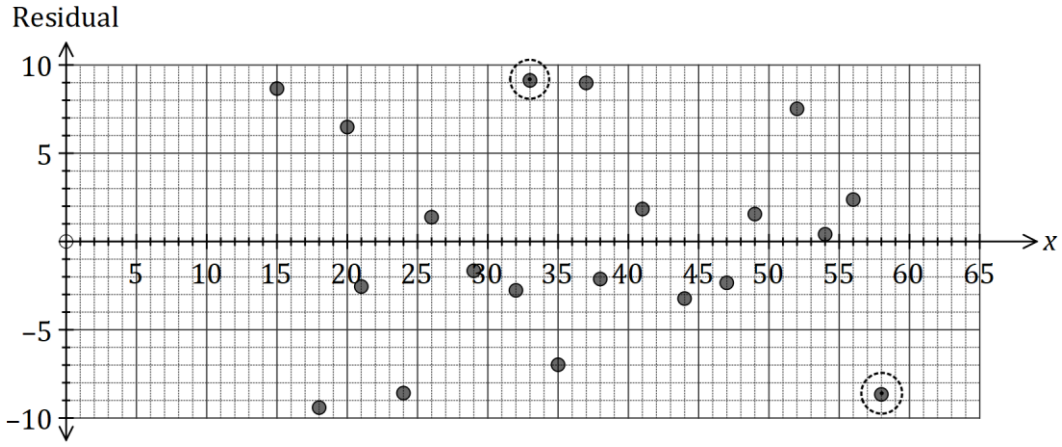
Solution
Yes, as the percentages in each column are quite different. For example, 10% of girls are underweight but only 4% of boys are.
Specific behaviours
✓ yes to association ✓ uses differences in column percentages to justify

See next page

Question 18 **(6 marks)**

A statistician wants to check whether a linear model is appropriate for a bivariate data set they are analysing. The least-squares line to model the linear relationship is $y = 1.54x - 13.9$ and the correlation coefficient between the variables is very strong.

The residual plot using the linear model is shown below for all but two of the data points.



(a) Calculate the residuals for the missing points $(33, 46)$ and $(58, 66.5)$ and plot them on the graph above. (4 marks)

Solution
$y(33) = 36.92$ and $46 - 36.92 = 9.08$ $y(58) = 74.42$ and $66.5 - 74.42 = -8.92$
Specific behaviours
✓ calculates both predicted values ✓ residual for one point ✓ residual for second point ✓ accurately plots both residuals

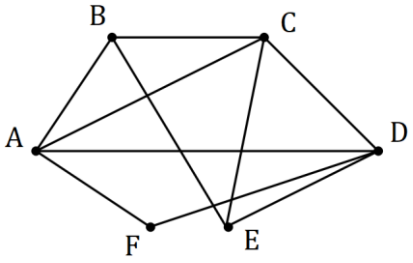
(b) Is fitting a linear model to the data appropriate? Explain. (2 marks)

Solution
Linear model is appropriate as no pattern is evident in the residuals.
Specific behaviours
✓ states linear model is appropriate ✓ refers to no pattern evident in residuals

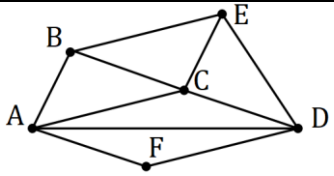
See next page

Question 19 **(7 marks)**

Each vertex on the graph below represents an airport and an edge between two airports indicates that an airline has a direct flight, in both directions, between the airports.



(a) Redraw the graph to clearly show that it is planar. (1 mark)

Solution

Specific behaviours
✓ correct equivalent, no crossed edges

(b) Demonstrate that the graph satisfies Euler's formula. (2 marks)

Solution
$v + f - e = 6 + 6 - 10 = 2$
Specific behaviours
✓ correct count of v, f and e
✓ substitutes into formula and simplifies correctly

In order to check in-flight catering quality, an airline manager plans to leave airport A, travel on at least one flight between the 10 pairs of airports and then return to A. The manager does not use any other mode of transport between airports.

(c) Determine the minimum number of flights the manager must take and list, in order, the airports visited. (2 marks)

Solution
$A - F - D - A - C - D - E - C - B - E - B - A = 11$ flights
Specific behaviours
✓ correct number
✓ correct list (many exist)

(d) Another manager, based at a different airport, claimed they could carry out the quality check in fewer flights by starting and finishing at their airport. Comment on this claim. (2 marks)

Solution
Claim is true if manager based at B or E. Graph is semi-Eulerian and starting from these airports only 10 flights required.
Specific behaviours
✓ agrees with claim conditionally
✓ states correct number of flights

See next page

Question 20 **(8 marks)**

(a) An investor has $\$540$ in an account. One month later, and at the start of each subsequent month, a deposit of $\$35$ is added to the account. Interest, calculated as 0.72% of the balance at the start of the month, is added to the account just before each deposit is made.

The account balance after n deposits is T_n, and can be modelled by the recurrence relation $T_{n+1} = 1.0072T_n + 35,\ T_0 = 540$.

(i) Determine the balance in the account after 4 deposits have been made. (1 mark)

Solution
$T_4 = \$697.24$
Specific behaviours
✓ correct amount

(ii) After how many deposits does the balance of the account first exceed $\$1\,200$ and what is the balance of the account at that time. (2 marks)

Solution
$T_n > 1200 \Rightarrow n = 17$ $T_{17} = \$1\,240.58$
Specific behaviours
✓ correct month; ✓ correct amount

(b) The investor also has $\$85$ in another account. One week later, and at the start of each subsequent week, a deposit of $\$5.75$ is added to the account. Interest, calculated as 0.095% of the balance at the start of the week, is added to the account just before each deposit is made.

(i) Write a recurrence relation to model the balance of this account after n deposits. (3 marks)

Solution
$T_{n+1} = 1.00095T_n + 5.75, \quad T_0 = 85$
Specific behaviours
✓ correct multiplier; ✓ correct addition; ✓ correct initial term

(ii) Determine the balance in this account after 40 deposits have been made. (1 mark)

Solution
$T_{40} = \$322.60$
Specific behaviours
✓ correct amount

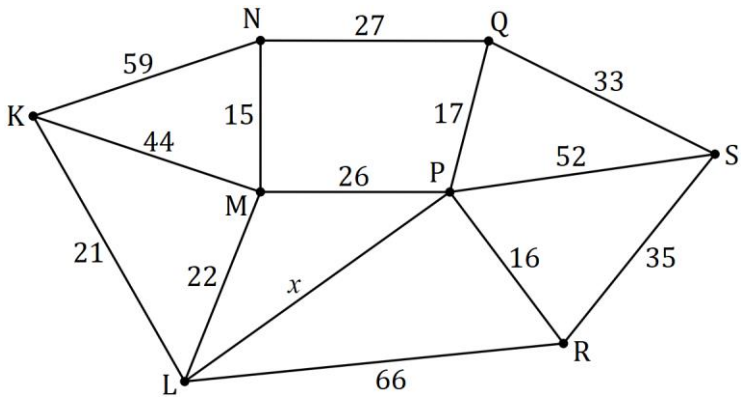
(iii) By considering the total deposits made, or otherwise, determine the total interest added to this account after 40 deposits have been made. (1 mark)

Solution
$\text{Deposits} = 40 \times 5.75 = 230$ $\text{Interest} = 322.60 - 85 - 230 = \$7.60$
Specific behaviours
✓ correct amount

See next page

Question 21 **(8 marks)**

The vertices below represent 8 computers in a network and the weights on each edge represent the time, in milliseconds, for a signal to be sent directly between connected computers.



(a) Given that $x = 49$, determine the path required and the time taken to send a signal in the least time between

(i) L and R. (2 marks)

Solution
$L - M - P - R$ in $22 + 26 + 16 = 64$ ms
Specific behaviours
✓ correct path; ✓ correct time for stated path

(ii) K and P. (2 marks)

Solution
$K - L - M - P$ in $21 + 22 + 26 = 69$ ms
Specific behaviours
✓ correct path; ✓ correct time for stated path

(iii) K and S. (2 marks)

Solution
$K - L - M - N - Q - S$ in $21 + 22 + 15 + 27 + 33 = 118$ ms
Specific behaviours
✓ correct path; ✓ correct time for stated path

(b) Determine the largest value of x, to the nearest millisecond, to ensure that the fastest route to send a signal between K and S will pass through P. Justify your answer. (2 marks)

Solution
$K - L - P - Q - S$ in $21 + x + 17 + 33 = 71 + x$ $71 + x < 118$ and so largest value of $x = 46$
Specific behaviours
✓ explanation ✓ correct value of x

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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